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# Control-oriented thermal management of solid oxide fuel cells based on a modified Takagi–Sugeno fuzzy model

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ABSTRACT

Thermal management for a solid oxide fuel cell (SOFC) is actually temperature control, due to the importance of cell temperature for the performance of an SOFC. An SOFC stack is a nonlinear and multi-variable system which is difficult to model by traditional methods. A modified Takagi–Sugeno (T–S) fuzzy model that is suitable for nonlinear systems is built to model the SOFC stack. The model parameters are initialized by the fuzzy c-means clustering method, and learned using an off-line back-propagation algorithm. In order to obtain the training data to identify the modified T–S model, a SOFC physical model via MATLAB is established. The temperature model is the center of the physical model and is developed by enthalpy-balance equations. It is shown that the modified T–S fuzzy model is sufficiently accurate to follow the temperature response of the stack, and can be conveniently utilized to design temperature control strategies.



## 1. Introduction

The solid oxide fuel cell (SOFC), operated at temperatures between 600 and 1000 °C, is a promising technology for sustainable energy conversion, having a wide range of possible applications. Its unique advantages are high efficiency, fuel flexibility, and low pollutant emissions, while combined heat and power generation can be realized with the waste heat produced in SOFCs [1]. However, the high operating temperature also brings serious challenges to the technology, such as material selection, thermal management, reliability and durability. Thermal management associated with temperature and its distribution is a critical factor for the cell performance and stack behavior, affecting cell integrity, cell voltage, current density distribution and power output [2,3].

An SOFC is a nonlinear multi-variable system which is difficult to model by traditional methodologies. In order to effectively control the cell temperature in terms of control strategies, it is necessary to develop model-based control methods. In recent decades, mathematical models have been established, emphasizing the accurate description of cell internal mechanisms, such as mass- and energy-balances and electrochemical kinetics [3–8]. These models are useful for cell design, analysis and optimization. However, they are complicated for use as model-based control methods [9], and in some cases they are impracticable. In addition, researchers have tried to establish models via data-driven approaches so as to obtain valid control strategies [10,11]. To control the complex SOFC system, simpler data-driven models based on system identification methodology are useful and convenient. Arriagada presented an artificial neural network to identify the SOFC model, which was trained by experimental data for prediction of various SOFC operational parameters. This method can accomplish performance analysis and prediction, but it is not perfect for the control purpose, and the experimental data may not sufficiently reflect all the characteristics of an SOFC [12]. Jurado [13,14] proposed a Hammerstein model for identification and predictive control. This model is specifically suitable for nonlinear systems where the nonlinear block is static and followed by a linear system, and can only be applied for small signal and transient stability studies. Huo et al. [15] and Kang et al. [16] used the least squares support vector machine to establish a data-driven model for an SOFC, achieved a solution by solving a set of linear equations. This model, based on mass- and energy-balance equations, describes the temperature dynamics and predicts stack voltage.

Fuzzy modeling and identification based on measured data are effective tools for the approximation of an uncertain nonlinear system [17]. Among various fuzzy modeling methods, the Takagi–Sugeno (T–S) fuzzy model has attracted most attention [18] due to it having fewer rules than other models. Each rule

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**Nomenclature**

| | |
|---|---|
| $A$ | cell active area ($m^2$) |
| $C_p$ | specific heat capacity of gas species ($J\,mol^{-1}\,K^{-1}$) |
| $\dot{C}_p$ | area specific heat capacity ($J\,K^{-1}\,m^{-2}$) |
| $F$ | Faraday's constant (96,485 $C\,mol^{-1}$) |
| $h$ | specific enthalpy ($J\,mol^{-1}$) |
| $H$ | enthalpy (J) |
| $\dot{H}$ | enthalpy flow (W) |
| $i$ | current density ($A\,m^{-2}$) |
| $i_0$ | exchange current density ($A\,m^{-2}$) |
| $i_r$ | reaction current density ($A\,m^{-2}$) |
| $I$ | current (A) |
| $m$ | mass (kg) |
| $N$ | molar flux ($mol\,m^{-2}\,s^{-1}$) |
| $n$ | number of moles (mol) |
| $\dot{n}$ | molar flow ($mol\,s^{-1}$) |
| $n_i$ | molar number of species $i$ (mol) |
| $\dot{n}^c$ | hydrogen combustion molar flow ($mol\,s^{-1}$) |
| $P$ | pressure (bar) |
| $R$ | universal gas constant (8.314 $J\,mol^{-2}\,K^{-2}$) |
| $t$ | time (s) |
| $T$ | temperature (K) also period (s) |
| $T_0$ | ambient temperature (K) |
| $U$ | voltage (V) |
| $V$ | volume ($m^2$) |
| $x$ | input vector |
| $x_i$ | molar fraction of species |
| $y$ | output vector |

*Greek letters*

| | |
|---|---|
| $\alpha$ | symmetry factor learning factor |
| $\beta$ | a T–S factor |
| $\eta$ | overpotential (V) |
| $\nu$ | stoichiometric coefficient |

*Subscripts*

| | |
|---|---|
| $i$ | gas species |
| $r$ | reaction |

*Superscripts*

| | |
|---|---|
| $in$ | fuel cell inlet |
| $out$ | fuel cell outlet |

consequence with a linear function can be described by input–output mapping in a wide range and the fuzzy implication used in the model is relatively simple [19]. The T–S fuzzy model has the capability to approximate a large class of static and dynamic nonlinear systems [20,21], and has been successfully proved to be applicable in model identifications and control strategies [22–24]. In most previous applications, single-input/single-output or multi-input/single-output was considered [25–27]. Yang et al. [28] described a dynamic T–S fuzzy model of a molten carbonate fuel cell, consisting of voltage and temperature submodels, but the temperature submodel can only be valid within a relatively narrow temperature range. The modeling of SOFCs using the T–S model has not been reported so far.

In this study, a control-oriented thermal management of an SOFC stack using a modified T–S fuzzy model was conducted with an identification method. The training data were extracted from an SOFC stack physical model that was constructed with MATLAB. The physical model built by mass- and energy-balance equations, and enthalpy-balance equations can serve as a real SOFC stack. The temperature subsystem based on enthalpy-balance equations can realize temperature control strategy. This T–S fuzzy model consists of if-then rules with fuzzy antecedents and mathematical functions in the consequent part. Unlike the original T–S model, SOFC input and output variable spaces were divided into many subspaces according to the principle of the fuzzy c-means (FCM) clustering method [29]. The model parameters were learned by using the back-propagation (BP) learning algorithm [30]. The task of system identification was to determine both the nonlinear parameters of antecedents and the linear parameters of the rule consequence. The satisfactory results show that the modified T–S fuzzy model can achieve the temperature control strategy.

## 2. SOFC physical model

In this SOFC physical model, some simplifications and assumptions are made, since high accuracy is not necessary for a physical model that serves for a subsequent control strategy. Any deviations between the model and the real fuel cell can be managed by a feedback loop in the control system [31]. The proposed SOFC stack physical model has the following assumptions:

(1) Stack is fed with hydrogen and air; the fuel processor dynamics are not included.
(2) A uniform gas distribution among the cells is assumed, since there is a small deviation of the gas distribution.
(3) There is no heat transfer among the cells. Each cell has the same temperature and current density.
(4) There is no heat exchange between the stack and the ambient environment.
(5) The channels that transport gases along the electrodes have a fixed volume and small dimension, so there is a constant pressure in the stack.

### 2.1. Mass-balance model

For a generic species $i$, the dynamic mole-balance is:

$$\frac{dn_i}{dt} = \dot{n}^{in}x_i^{in} - \dot{n}^{out}x_i + \nu_i\frac{i_rA}{F} \tag{1}$$

The stoichiometric factor $\nu_i$ indicates how many moles of the species are produced or consumed for each mole of electrons transferred. The anodic and cathodic reactions in the SOFC are, respectively:

$$H_2 + O^{2-} \rightarrow H_2O + 2e^-$$

$$(1/2)O_2 + 2e^- \rightarrow O^{2-} \tag{2}$$

then $\nu_{O_2} = -1/2$, $\nu_{H_2} = -1$, $\nu_{H_2O} = 1$, $\nu_{N_2} = 0$.
According to the Butler–Volmer equation [32]:

$$i_r = i_0(e^{\alpha(nF/RT)\eta} - e^{-(1-\alpha)(nF/RT)\eta}) \tag{3}$$

When pressure is constant, the total outlet molar flow depends only on the transient reaction rate and temperature and can be calculated using the following relation:

$$\dot{n}^{out} = \dot{n}^{in} + \sum_i \nu_i\frac{i_rA}{F} + \frac{pV}{RT^2}\frac{dT}{dt} \tag{4}$$

The term $\sum_i \nu_i(i_rA/F)$ represents the algebraic sum of gas reaction molar flow, and $(pV/RT^2)(dT/dt)$ reflects thermal expansion. Inserting Eq. (4) into Eq. (1) and assuming constant pressure, the molar-balance becomes:

$$\frac{pV}{RT}\frac{dx_i}{dt} = \dot{n}^{in}(x_i^{in} - x_i) + \frac{i_rA}{F}(\nu_i - x_i\sum_j \nu_j) \tag{5}$$

### 2.2. Energy-balance model

Energy-balance is a very important aspect for SOFCs operated at high temperature. Here only the enthalpy-balance is considered. The main sources and sinks of heat for an SOFC are entering and exiting flow, the heat generated by cell reactions, and the heat lost to the environment. An enthalpy-balance equation yields:

$$\frac{dH}{dt} = \dot{H}^{in} - \dot{H}^{out} - i_r VA - \dot{H}^{loss} \tag{6}$$

The term $i_r VA$ represents the electrical power generated. The relationship between enthalpy and temperature can be expressed as

$$\frac{dH}{dt} = A\dot{c}_p \frac{dT}{dt} \tag{7}$$

The entering enthalpy flow also contains the heat generated by the combustion of hydrogen, hence the total entering enthalpy flow is equal to the enthalpy associated with the ambient air plus the

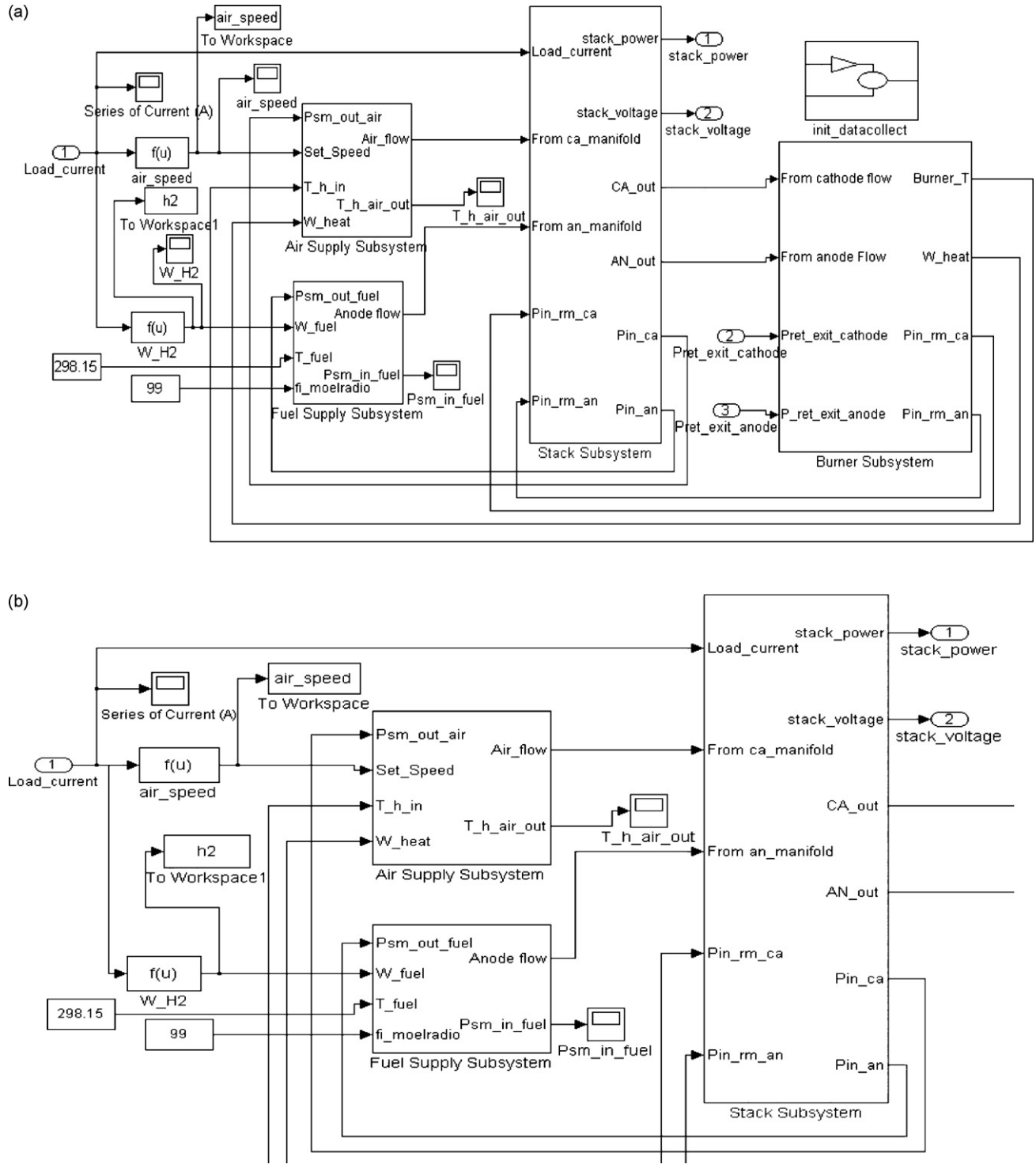


**Fig. 1.** MATLAB diagram of air and $H_2$ flowing into the Stack Subsystem via the supply subsystem.

entering hydrogen flow:

$$\dot{H}^{in} = \dot{n}_{air} h_{air}(T_0) + (\dot{n}_{H_2} + \dot{n}^c_{H_2}) h_{H_2}(T_{H_2}) \quad (8)$$

In reality, $T_{H_2}$ may be significantly different from $T_0$, as hydrogen from various sources may undergo different processes, such as expansion, evaporation or dissociation; however, for simplification $T_{H_2} = T_0$ is assumed without causing any meaningful deviation.

The total outlet molar flow will be larger than or equal to the entering one and the expression of this is:

$$\dot{H}^{out} = \sum_i \dot{n}_i^{out} h_i(T) \quad (9)$$

and the outlet molar flow for species $i$, $\dot{n}_i^{out}$ can also be expressed as

$$\dot{n}_i^{out} = \dot{n}_i^{in} + \nu_i \left( \dot{n}^c_{H_2} + \frac{i_r A}{2F} \right) + x_i \frac{pV}{RT^2} \frac{dT}{dt} \quad (10)$$

where $\dot{n}_i^{in}$ is the entering molar flow, $\nu_i(\dot{n}^c_{H_2} + (i_r A/2F))$ is the molar flow associated with hydrogen reaction and combustion, and the term $x_i\,(pV/RT^2)(dT/dt)$ is related to gas thermal expansion.

The heat loss to the environment, $\dot{H}^{loss}$, is generally a beneficial factor, as a fuel cell stack prefers heat to be removed from it. However, its actual expression depends on the stack materials and layout. For the sake of simplicity, it is assumed that $\dot{H}^{loss} = 0$ [31].

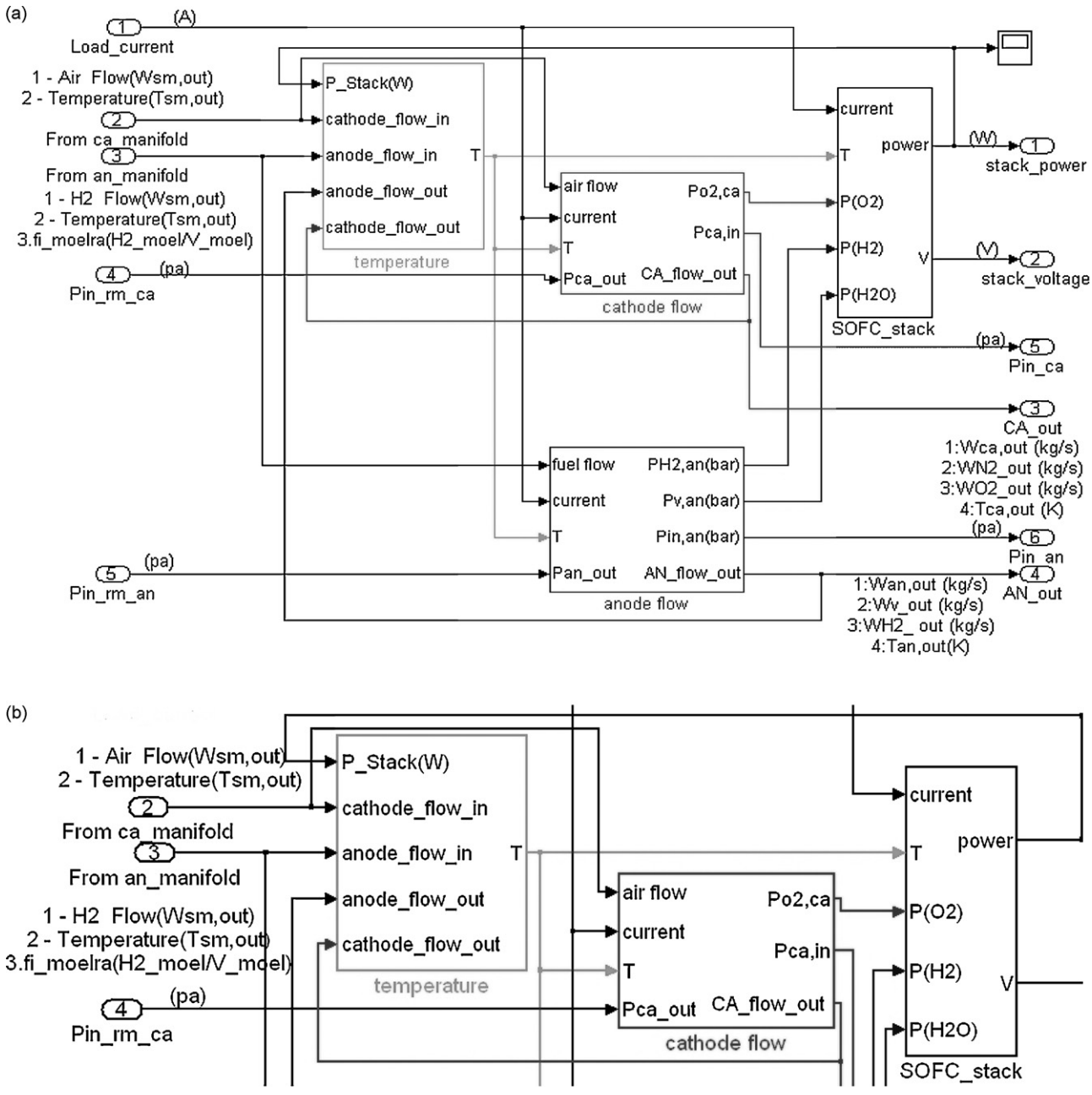


**Fig. 2.** MATLAB diagram showing air and $H_2$ flow from the ca_manifold and an_manifold into the temperature block.

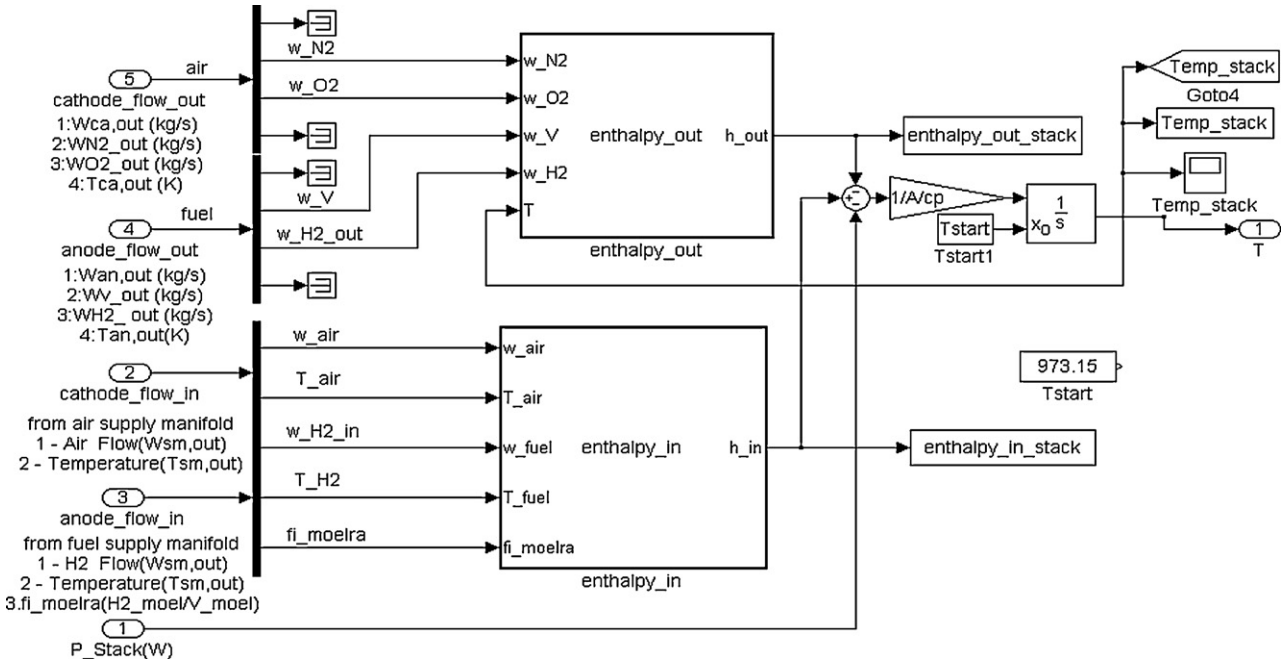


**Fig. 3.** The structure of the temperature model subsystem.

### 2.3. Temperature control dynamic model

The objective of temperature control is to keep the stack at a given temperature by manipulating the air flow. Having assumed that temperature is uniform in a stack, Eq. (6) determines its dynamics. According to simultaneous Eqs. (6) and (7), it can be obtained that:

$$A\dot{c}_p\frac{dT}{dt} = \dot{H}^{in} - \dot{H}^{out} - i_r VA$$
$$= (\dot{H}^{in}(T) - \dot{H}^{out}(T)) - (\dot{H}^{in}(T) - \dot{H}^{in}(T_0)) - i_r VA \quad (11)$$

The terms $(\dot{H}^{in}(T) - \dot{H}^{out}(T))$ and $(\dot{H}^{in}(T) - \dot{H}^{in}(T_0))$, respectively, represent the reaction heat and the sensible heat.

In Eq. (11) it assumed that the specific heat capacity for all the species is approximately the same on condition that the pressure is constant. This is adequate for the control-oriented modeling. According to Eq. (11), the temperature control model can be built with MATLAB.

### 2.4. Temperature control model realization in MATLAB

The stack temperature is influenced essentially by the entering flows and the reaction rate, and the reaction rate is determined according to the requirements of the electric utility. Therefore, the only remaining way to influence the stack's temperature is changing the entering flows. This is the standard operating mode, where the entering flows are manipulated to approach the stack reference temperature. This physical MATLAB model has two input variables, air flow and $H_2$ flow, and one output variable, $T$. Once more variables are selected from the physical model and more training data are obtained for the modified T–S model, the multi-input (load current, air flow hydrogen flow rate, etc.) and multi-output (stack temperature, stack voltage stack power, etc.) system can be established.

A virtual 3.5 kW SOFC stack is used in the simulation. The stack consists of 60 cells with anode and cathode gases in cross-flow and cell active area 0.01 $m^2$. As shown in Figs. 1–3, the physical model replaces the real SOFC stack to generate the simulation data required for the modified T–S fuzzy model. In Fig. 1, the input variables are air flow and $H_2$ flow. Air and $H_2$, respectively, enter into the Air Supply Subsystem block and the Fuel Supply Subsystem block. The internal structure of the Stack Subsystem block is shown in Fig. 2. The Stack Subsystem block includes a cathode flow block, anode flow block, stack block and temperature block. Eq. (11) is built in the temperature block shown in Fig. 3; the stack temperature (Temp_stack) can be obtained according to Eq. (11).

The enthalpy expressions can be acquired from the published literature [33] and the relationship between enthalpy and temperature for different gases is reflected in these expressions. The Enthalpy_out block and Enthalpy_in block can be expressed as follows ($w$ expresses molar flow):

- For Enthalpy_in block:

$$h\_in = w\_\text{air} \times h\_\text{air} + w\_H_2 \times h\_H_2 + w\_V \times h\_V \quad (12)$$

where

$$\begin{aligned} h\_\text{air} &= -1.0947e4 + 32.50 \times T\_\text{air} \\ h\_H_2 &= -0.9959e4 + 30.73 \times T\_\text{fuel} \\ h\_V &= -25.790e4 + 42.47 \times T\_\text{fuel} \end{aligned} \quad (13)$$

$$\begin{aligned} w\_H_2 &= \frac{w\_\text{fuel} \times fi\_\text{moelra} \times 2.016}{(fi\_\text{moelra} \times 2.016 + 18.02)} \\ w\_V &= w\_\text{fuel} - w\_H_2 \end{aligned} \quad (14)$$

- For the Enthalpy_out block:

$$h\_\text{out} = w\_O_2 \times h\_O_2 + w\_N_2 \times h\_N_2 + w\_H_2 \times h\_H_2 + w\_V \times h\_V \quad (15)$$

and

$$\begin{aligned} h\_N_2 &= -1.0590e4 + 31.40 \times T \\ h\_H_2 &= -0.9959e4 + 30.73 \times T \\ h\_O_2 &= -1.2290e4 + 35.12 \times T \\ h\_V &= -25.790e4 + 42.47 \times T \end{aligned} \quad (16)$$

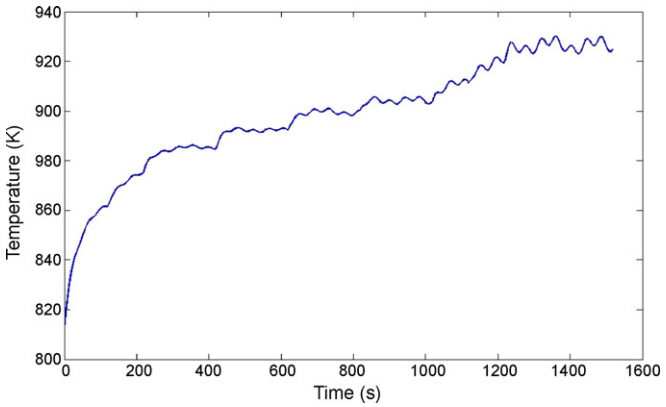


**Fig. 4.** The temperature response of the SOFC stack physical model.

Based on the physical model, it can be established that the load signal change can vary the SOFC stack temperature. In addition, the stack temperature also fluctuates with the input variables (air flow and $H_2$ flow). As shown in Fig. 4, the temperature of the SOFC stack fluctuates between 815 and 925 K with an ascending trend and the temperature curve is fluctuating rather than smooth. Sharp fluctuations of stack temperature present difficulties for temperature control, and therefore a data-driven model, instead of a physical model, is used to carry out the control strategy. For this purpose, a modified T–S model is applied with data provided by the above physical model.

## 3. A modified T–S fuzzy model of the structure and algorithm

### 3.1. The structure of T–S fuzzy model

An SOFC system can be considered as a multi-input/multi-output system that can be divided into $q$ (number of input variables) multi-input and single-output (MISO) systems, and each MISO system can be fitted by a fuzzy T–S model independently. The discrete-time system can be modeled by the collection of c fuzzy rules. It has been theoretically proved that the T–S model has the capability of closely approximating a real system by increasing the number of fuzzy rules; therefore, the model is highly precise.

The modified T–S model presented here is a fuzzy logic system which is jointly composed of the fuzzy rule base and weight mean defuzzier.

The fuzzy rule is in the form of "if. . .then. . .". The $i$th rule of the $l$th output $\hat{y}_{l,i}(k+1)$ is given by

$$\begin{aligned} &R_{l,i}: \textit{If } x(k) \textit{ is } A_i^l, \textit{ then} \\ &\hat{y}_{l,i}(k+1) = p_{i,0}^l + p_{i,1}^l x_{k1} + \cdots + p_{i,n}^l x_{kn} \quad (i = 1, \cdots, c) \end{aligned} \tag{17}$$

where $c$ is the number of rules, $A_i^l = \left\{A_{i,1}^l, \cdots, A_{i,n}^l\right\}$ is the set of membership functions associated with the $i$th rule, and $p_i^l = \left[p_{i,0}^l, p_{i,1}^l, \cdots, p_{i,n}^l\right]$ is the parameter vector of the $i$th submodel. It is defined that:

$$\left\{u_j(k)\right\}_0^{nuj} = \left[u_j(k), \cdots, u_j(k - nuj + 1)\right] \quad (j = 1, \cdots, n_j)$$

$$\{y_l(k)\}_0^{nyl} = [y_l(k), \cdots, y_l(k - nyl + 1)] \quad (l = 1, \cdots, q)$$

$nuj$ and $nyl$ are the order of the input variable $u_j$ and output variable $y_l$, respectively, and each MISO subsystem can be denoted as

$$y_l(k+1) = f[x(k)] \quad l = 1, \cdots, q \tag{18}$$

where $x(k) = \left\{[u_1(k)]_0^{nu1}, \cdots, \left[u_{nj}\right]_0^{nuj}, [y_1(k)]_0^{ny1}, \cdots, \left[y_q(k)\right]_0^{nyq}\right\} = [x_{k1}, \cdots, x_{kn}]$ is the regression data vector consisting of input–output data at the $k$th instant and before.

The antecedent part of "If. . ." in the form of the fuzzy set is equal to the fuzzy partition of data space of the SOFC input–output variables, and the consequent part of "then. . ." represented by a linear functional relation is a linear composition of the input–output variables. The identification is to find the fuzzy SOFC model with reasonable complexity to minimize the error between the fuzzy SOFC model output and the real SOFC stack output. In this model, what is to be partitioned is the whole space spanned by regression data vectors, but not the single variable space presented by Takagi and Sugeno [18].

Membership function of the fuzzy set $A_j$ can be defined as the product of Gaussian functions:

$$\mu_{Ai}(x_m) = \prod_{p=1}^{n} \left\{ \exp\left[ -\frac{1}{2} \left( \frac{x_{mp} - \upsilon_{ip}}{\sigma_{ip}} \right)^2 \right] \right\} \tag{19}$$

where $x_m$ is the $m$th regression data vector, $p(=1, \ldots, n)$ is the dimension of data vector, $\upsilon_i=[\upsilon_{i1}, \upsilon_{i2}, \ldots, \upsilon_{in}]$ and $\sigma_i=[\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{in}]$ are subsequent identification parameters of the membership functions. According to the above simultaneous equations, the fuzzy reasoning output can be expressed as

$$\begin{aligned} y_l(k+1) &= \frac{\sum_{i=1}^{c} w_i \hat{y}_{l,i}(k+1)}{\sum_{i=1}^{c} w_i} \\ &= \frac{\sum_{i=1}^{c} w_i (p_{i,0}^l + p_{i,1}^l x_{k1} + \cdots + p_{i,n}^l x_{kn})}{\sum_{i=1}^{c} w_i} \\ &= \frac{\sum_{i=1}^{c} w_i ([\,1 \quad x_m\,] \times p_i^T)}{\sum_{i=1}^{c} w_i} = \frac{\phi_1}{\phi_2} \end{aligned} \tag{20}$$

with

$$w_i = \prod_{p=1}^{n} \left\{ \exp\left[ -\frac{1}{2} \left( \frac{x_{mp} - \upsilon_{jp}}{\sigma_{jp}} \right)^2 \right] \right\} \tag{21}$$

where $[p_{1,0}^l \cdots p_{c,0}^l \cdots p_{1,1}^l \cdots p_{c,1}^l \cdots p_{1,n}^l \cdots p_{c,n}^l]^T$ is the subsequent identification parameter vectors of the consequent part, and $w_i$ is the match degree of $m$th data vector corresponding to $i$th rules.

### 3.2. Model parameters of BP learning algorithm

The above fuzzy logic system can be represented as a feed-forward network in Fig. 5, whose weight coefficients have clear physical meanings and can be learned by back-propagation of the output predictive error [23,30]. A parameter set of the fuzzy system that minimizes the output predictive error can be found through a collection of training data produced by the physical SOFC model:

$$E = \frac{\sum_{m=1}^{q} (y_{l,m} - \hat{y}_{l,m})^2}{2} \tag{22}$$

where $y_{l,m}$ is the SOFC physical model output of $m$th data vector, and $\hat{y}_{l,m}$ is predictive output of fuzzy inference system. Calculating the gradient of predictive output error $E$ through Eqs. (21) and (22) can be used to obtain the learning algorithm of fuzzy inference system parameter vectors $\upsilon_i$, $\sigma_i$ and $p_i$:

$$\upsilon_{i,p}(k+1) = \upsilon_{i,p}(k) - \alpha \frac{\partial E}{\partial \upsilon_{i,p}}(k) \tag{23}$$

$$\sigma_{i,p}(k+1) = \sigma_{i,p}(k) - \alpha \frac{\partial E}{\partial \sigma_{i,p}}(k) \tag{24}$$

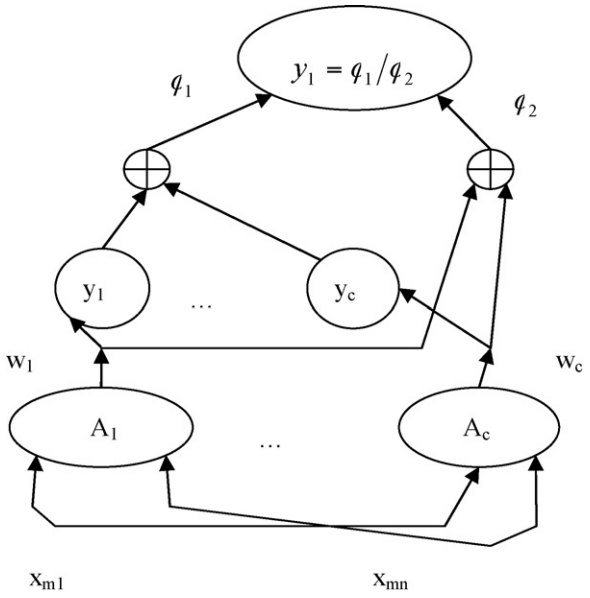


**Fig. 5.** Feed-forward network form of fuzzy logic system.

$$p_{i,0}^{l}(k+1) = p_{i,0}^{l}(k) - \alpha \frac{\partial E}{\partial p_{i,0}^{l}}(k) \tag{25}$$

$$p_{i,p}^{l}(k+1) = p_{i,p}^{l}(k) - \alpha \frac{\partial E}{\partial p_{i,p}^{l}}(k) \tag{26}$$

where $\alpha$ is the learning factor, and $p = 1, \ldots, n$.

### 3.3. The initialization of antecedent parameters

Since the BP algorithm tends to fall into localized optimization, the initialization of the design parameter is very important. In this study, the FCM clustering method [29] is employed to initialize the antecedent parameter vectors $\upsilon_i$ and $\sigma_i$:

$$\upsilon_i = c_i \tag{27}$$

$$\sigma_i = \beta \times \frac{\sum_{i=1}^{q}(U_{im} \times \left\|x_m - c_i\right\|)}{\sum_{i=1}^{q} U_{im}} \quad \text{for } i = 1, \cdots, c \tag{28}$$

where $\beta$(=3) is a given factor in advance, the first set of $c$ data vectors are obtained from the training data set as the initialization parameters. The FCM initialization method is more likely to avoid local optimization of the BP algorithm.

### 3.4. On-line adjustment of consequent parameters

For a complex system that operates over a wide range of conditions, it is difficult to adopt an off-line model to accurately approximate the dynamic characteristics of the system. In the proposed algorithm, if the predictive precision of the model cannot satisfy the need for real-time control, the consequent parameters $p_i^l$ should be rectified on-line.

Equation (20) can also be expressed as

$$\hat{y}_l(k+1) = \frac{\sum_{i=1}^{c} w_i[x(k)]\hat{y}_{l,i}(k+1)}{\sum_{i=1}^{c} w_i[x(k)]} = \sum_{i=1}^{c} \mu_i(k) \times \hat{y}_{l,i}(k+1) = \Phi(k) \times \theta \tag{29}$$

**Table 1**
Parameters of the SOFC stack used in the physical model.

| Item | Value |
|---|---|
| Cell number | 60 |
| Cell active area ($m^2$) | 0.01 |
| The heat capacity per cell area ($J K^{-1} m^{-2}$) | 7000 |
| The arm power (kW) | 3.5 |
| Anode total volume ($m^3$) | 0.005 |
| Cathode total volume ($m^3$) | 0.001 |
| Gas inlet flow constant | $0.3629 \times 10^{-5}$ |
| Anode gas outlet flow constant | $0.0544 \times 10^{-5}$ |
| Cathode gas outlet flow constant | $0.2118 \times 10^{-5}$ |
| Air heat capacity ($J kg^{-1} K^{-1}$) | 1004 |
| Air density ($kg m^{-3}$) | 1.23 |
| Oxygen density ($kg m^{-3}$) | 1.36 |
| Nitrogen density ($kg m^{-3}$) | 1.19 |
| Air gas constant ($kg m^{-3}$) | 286.9 |
| Oxygen gas constant ($kg m^{-3}$) | 259.8 |
| Nitrogen gas constant ($kg m^{-3}$) | 296.8 |
| Hydrogen gas constant ($kg m^{-3}$) | $4.1243 \times 10^3$ |
| Oxygen molar mass ($kg mol^{-1}$) | 0.032 |
| Nitrogen molar mass ($kg mol^{-1}$) | 0.028 |
| Hydrogen molar mass ($kg mol^{-1}$) | $2.016 \times 10^{-5}$ |
| Faraday's constant | 96485.34 |
| Universal gas constant ($kg m^{-3}$) | 8.31451 |

with $\mu_i(k) = \frac{w_i[x(k)]}{\sum_{i=1}^{c} w_i[x(k)]}$

$$\Phi(k) = [\mu_1, \mu_2, \cdots, \mu_c, \mu_1 x_k, \mu_2 x_k, \cdots, \mu_c x_k, \cdots, \mu_1 x_{kn}, \cdots, \mu_c x_k]$$

$$\theta = \left[p_{1,0}^{l} \cdots p_{c,0}^{l}\, p_{1,1}^{l} \cdots p_{c,1}^{l} \cdots p_{1,n}^{l} \cdots p_{c,n}^{l}\right]^T$$

where $w_i[x(k)]$ is the match degree of $m$th data vector $x(k)$ with respect to $A_i^l$. Then the consequent parameters $\left[p_{1,0}^{l} \cdots p_{c,0}^{l} \cdots p_{1,1}^{l} \cdots p_{c,1}^{l} \cdots p_{1,n}^{l} \cdots p_{c,n}^{l}\right]^T$ are identified by the least squares method.

The modified algorithms significantly improve the accuracy of the identification and ensure a more accurate model to fit the dynamic characteristics of the SOFC stack. It is not necessary to consider the SOFC materials, structures and other performance parameters in the mechanism model and determination of these factors is more complicated. Clearly, the modified fuzzy model is simple and convenient to build, as long as the input–output test data are available. A multi-variable model can be readily established by the fuzzy modeling methods. In addition, after fuzzy identification, the fuzzy model can facilitate the application of an SOFC automatic control system.

## 4. Results

### 4.1. Preparation of training data

In this experiment, the SOFC system is regarded as a two-input and single-output system. The detailed parameters used for the physical stack model are given in Table 1. From the physical SOFC model testing experiments, training data of different air and hydrogen flow speeds corresponding to the SOFC stack temperature can be obtained, which forms a data group that is divided further into a training group and a testing group. The training group is used to identify the modified T–S fuzzy model; and the testing group is used to validate the modified T–S fuzzy model.

### 4.2. Identification result of modified T–S fuzzy model

The training data are classified into 12 groups using the FCM clustering method, and the results of the FCM clustering method are employed to initialize the antecedent parameters. In the BP learning algorithm, the exponent $m$ ($m$ = 2) is a real number, $c$ ($c$ = 12) is the

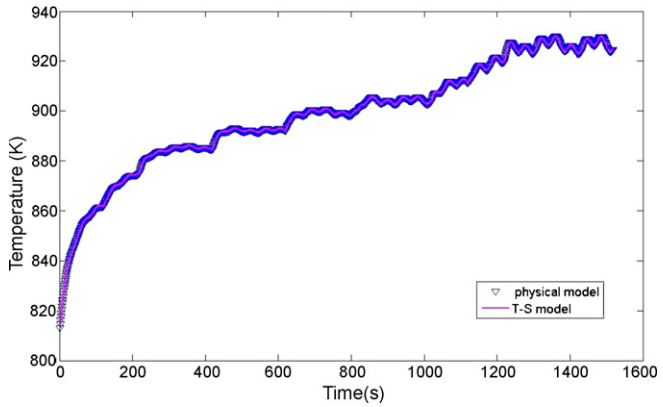


**Fig. 6.** Comparison of temperature responses of SOFC stack generated by the physical model and the modified T–S model.

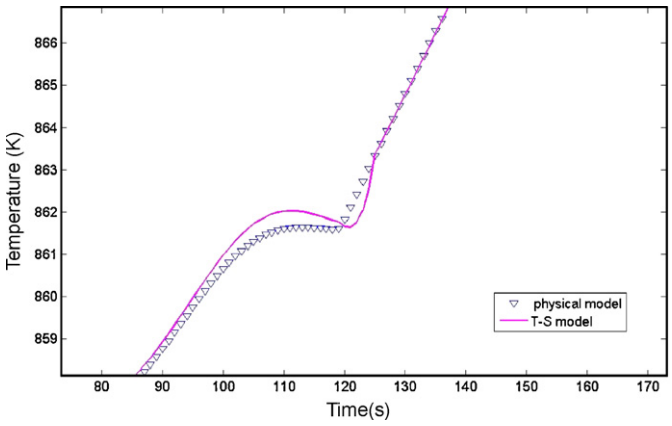


**Fig. 7.** Early stage magnification of the temperature responses shown in Fig. 6.

number of fuzzy rules, $e$ ($e = 0.001$) is the stop criterion, the learning factor is 0.01, and the learning number is 100; as a consequence, an off-line T–S fuzzy SOFC model, which consists of 12 fuzzy rules, is obtained. The comparison of stack temperature dynamic curves between the data-driven model (modified T–S fuzzy model) and the physical model (simulation stack model) is represented in Fig. 6, with the early stage shown in Fig. 7. The temperature deviation curve is shown in Fig. 8. Less than ±0.6 K is achieved, suggesting that the modified T–S fuzzy model can approximate the dynamic behavior of the physical SOFC model with high accuracy and provide a satisfactory control strategy.

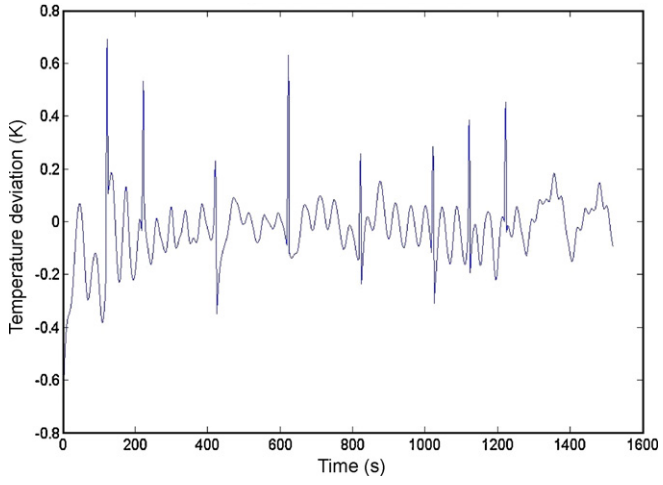


**Fig. 8.** The temperature deviation between results from the SOFC stack physical model and the modified T–S model.

## 5. Conclusions

In order to facilitate valid control-oriented modeling, a nonlinear modeling study of an SOFC stack using the modified T–S fuzzy model was carried out. It has been shown that the physical model, which is constructed from mass-balance and energy-balance equations with MATLAB, can be used to describe the SOFC stack and produce training data without conducting experiments. The temperature submodel is the center of the physical model and is developed by enthalpy-balance equations. Based on the data-driven approach, the modified T–S fuzzy model is more attractive because it avoids the use of complicated differential equations to build the SOFC model and quickly identifies the characteristics of the input–output relationship. The simulation results show that the modified T–S model yields a high accuracy over a relatively wide operating temperature range. If sufficient data from a real system are obtained, an optimized control-oriented SOFC stack model can be built automatically in the same way.

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